

CERN-PH-TH/2004-076

PROGRESS ON MODELS OF NEUTRINO MASSES AND MIXINGS

Guido Altarelli

CERN, Department of Physics, Theory Division
CH - 1211 Geneva 23

ABSTRACT

We review some recent results obtained by our group on models of neutrino masses and mixings in the general context of SUSY GUT's with an associated flavour symmetry

*Talk given at the Fujihara Seminar on
Neutrino Mass and Seesaw Mechanism, KEK (Japan), 23–25 February 2004*

CERN-PH-TH/2004-076
May 2004

1. Introduction

In this presentation I will describe some recent results obtained in collaboration with Ferruccio Feruglio and Isabella Masina. In the first part, based on ref. ¹⁾ recently updated including all available data, we discuss a quantitative study of the ability of models with different levels of hierarchy to reproduce the observed pattern of neutrino masses and mixings. As a flexible testing ground we consider models based on $SU(5) \times U(1)_F$. In this context, we have made statistical simulations of models with different patterns from anarchy to various types of hierarchy: normal hierarchical models with and without automatic suppression of the 23 (sub)determinant and inverse hierarchy models. We find that the hierarchical models have a significantly better success rate than those based on anarchy. The normal hierarchy models appear to maintain a considerable edge over inverse hierarchy or anarchy.

In the second part, based on ref. ²⁾, we discuss to which extent the observed mixing can arise from the diagonalisation of the charged lepton matrix. The neutrino mixing matrix U is in general of the form $U = U_e^\dagger U_\nu$, where U_e arises from the diagonalization of charged leptons and U_ν is from the neutrino sector. We consider the possibility that U_ν is nearly diagonal (in the lagrangian basis) and the observed mixing arises with good accuracy from U_e . We find that the fact that, in addition to the nearly maximal atmospheric mixing angle θ_{23} , the solar angle θ_{12} is definitely also large while at the same time the third mixing angle θ_{13} is small, makes the construction of a natural model of this sort considerably more complicated. We present an example of a natural model of this class. We also find that the case that U_ν is exactly of the bimaxing type is severely constrained by the bound on θ_{13} but not excluded. We show that planned experimental searches for θ_{13} could have a strong impact on bimaxing models.

2. Hierarchy versus Anarchy

The smallness of neutrino masses interpreted via the see-saw mechanism ³⁾ directly leads to a scale Λ for L non-conservation which is remarkably close to M_{GUT} . Thus neutrino masses and mixings should find a natural context in a GUT treatment of all fermion masses. The hierarchical pattern of quark and lepton masses, within a generation and across generations, requires some dynamical suppression mechanism that acts differently on the various particles. This hierarchy can be generated by a number of operators of different dimensions suppressed by inverse powers of the cut-off Λ_c of the theory. The different powers of $1/\Lambda_c$ may correspond to different orders in some symmetry breaking parameter v_f arising from the spontaneous breaking of a flavour symmetry. Here we describe some simplest models based on $SU(5) \times U(1)_F$

which illustrate these possibilities ⁴⁾. It is notoriously difficult to turn these models into fully realistic theories, due to well-known problems such as the doublet-triplet splitting, the proton lifetime, the gauge coupling unification beyond leading order and the wrong mass relations for charged fermions of the first two generations. Some of these problems can be solved by adopting the elegant idea of GUT's in extra dimensions ⁵⁾. Here we adopt the GUT framework simply as a convenient testing ground for different neutrino mass scenarios.

2.1. Models Based on Horizontal Abelian Charges

We discuss here some explicit examples of grand unified models in the framework of a unified SUSY SU(5) theory with an additional U(1)_F flavour symmetry ¹⁾. The SU(5) generators act “vertically” inside one generation, while the U(1)_F charges are different “horizontally” from one generation to the other. If, for a given interaction vertex, the U(1)_F charges do not add to zero, the vertex is forbidden in the symmetric limit. But the symmetry is spontaneously broken by the VEV's v_f of a number of “flavon” fields with non-vanishing charge. Then a forbidden coupling is rescued but is suppressed by powers of the small parameters v_f/Λ_c with the exponents larger for larger charge mismatch. We expect $M_{GUT} \lesssim v_f \lesssim \Lambda_c \lesssim M_{Pl}$. Here we discuss some aspects of the description of fermion masses in this framework.

In these models the known generations of quarks and leptons are contained in triplets Ψ_i^{10} and $\Psi_i^{\bar{5}}$, ($i = 1, 2, 3$) corresponding to the 3 generations, transforming as 10 and $\bar{5}$ of SU(5), respectively. Three more SU(5) singlets Ψ_i^1 describe the RH neutrinos. In SUSY models we have two Higgs multiplets H_u and H_d , which transform as 5 and $\bar{5}$ in the minimal model. The two Higgs multiplets may have the same or different charges. In all the models that we discuss the large atmospheric mixing angle is described by assigning equal flavour charge to muon and tau neutrinos and their weak SU(2) partners (all belonging to the $\bar{5} \equiv (l, d^c)$ representation of SU(5)). Instead, the solar neutrino oscillations can be obtained with different, inequivalent charge assignments. There are many variants of these models: fermion charges can all be non-negative with only negatively charged flavons, or there can be fermion charges of different signs with either flavons of both charges or only flavons of one charge. We can have that only the top quark mass is allowed in the symmetric limit, or that also other third generation fermion masses are allowed. The Higgs charges can be equal, in particular both vanishing or can be different. We can arrange that all the structure is in charged fermion masses while neutrinos are anarchical.

2.1.1. F(fermions) ≥ 0

Consider, for example, a simple model with all charges of matter fields being non-negative and containing one single flavon $\bar{\theta}$ of charge F = -1. For a maximum of

simplicity we also assume that all the third generation masses are directly allowed in the symmetric limit. This is realized by taking vanishing charges for the Higgses and for the third generation components Ψ_3^{10} , $\Psi_3^{\bar{5}}$ and Ψ_3^1 . If we define $F(\Psi_i^R) \equiv q_i^R$ ($R = 10, \bar{5}, 1$; $i = 1, 2, 3$), then the generic mass matrix m has the form

$$m = \begin{pmatrix} y_{11}\lambda^{q_1^R+q_1^{R'}} & y_{12}\lambda^{q_1^R+q_2^{R'}} & y_{13}\lambda^{q_1^R+q_3^{R'}} \\ y_{21}\lambda^{q_2^R+q_1^{R'}} & y_{22}\lambda^{q_2^R+q_2^{R'}} & y_{23}\lambda^{q_2^R+q_3^{R'}} \\ y_{31}\lambda^{q_3^R+q_1^{R'}} & y_{32}\lambda^{q_3^R+q_2^{R'}} & y_{33}\lambda^{q_3^R+q_3^{R'}} \end{pmatrix} v \quad , \quad (1)$$

where all the y_{ij} are dimensionless complex coefficients of order one and $m_u, m_d = m_l^T$, m_D and M arise by choosing $(R, R') = (10, 10), (\bar{5}, 10), (1, \bar{5})$ and $(1, 1)$, respectively. We have $\lambda \equiv \langle \bar{\theta} \rangle / \Lambda_c$ and the quantity v represents the appropriate VEV or mass parameter. The models with all non-negative charges and one single flavon have particularly simple factorization properties. For instance in the see-saw expression for $m_\nu = m_D^T M^{-1} m_D$ the dependence on the q_i^1 charges drops out and only that from $q_i^{\bar{5}}$ remains. In addition, for the neutrino mixing matrix U_{ij} , which is determined by m_ν in the basis where the charged leptons are diagonal, one can prove that $U_{ij} \approx \lambda^{|q_i^{\bar{5}} - q_j^{\bar{5}}|}$, in terms of the differences of the $\bar{5}$ charges, when terms that are down by powers of the small parameter λ are neglected. Similarly the CKM matrix elements are approximately determined by only the 10 charges ⁴⁾: $V_{ij}^{CKM} \approx \lambda^{|q_i^{10} - q_j^{10}|}$. If the symmetry breaking parameter λ is numerically close to the Cabibbo angle, we can choose:

$$(q_1^{10}, q_2^{10}, q_3^{10}) = (3, 2, 0) \quad , \quad (2)$$

thus reproducing $V_{us} \sim \lambda$, $V_{cb} \sim \lambda^2$ and $V_{ub} \sim \lambda^3$. The same q_i^{10} charges also fix $m_u : m_c : m_t \sim \lambda^6 : \lambda^4 : 1$. The experimental value of m_u (the relevant mass values are those at the GUT scale: $m = m(M_{GUT})$ ⁸⁾) would rather prefer $q_1^{10} = 4$. Taking into account this indication and the presence of the unknown coefficients $y_{ij} \sim O(1)$ it is difficult to decide between $q_1^{10} = 3$ or 4 and both are acceptable. Of course the charges $(q_1^{10}, q_2^{10}, q_3^{10}) = (2, 1, 0)$ would represent an equally good choice, provided we appropriately rescale the expansion parameter λ . Turning to the $\bar{5}$ charges, if we take (16, 6, 7, 9, 10)

$$(q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) = (b, 0, 0) \quad b \geq 0 \quad , \quad (3)$$

together with eq. (2) we get the patterns $m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \lambda^{3+b} : \lambda^2 : 1$. Moreover, the 22, 23, 32, 33 entries of the effective light neutrino mass matrix m_ν are all $O(1)$, thus accommodating the nearly maximal value of s_{23} . The small non diagonal terms of the charged lepton mass matrix cannot change this. We obtain, where arbitrary $o(1)$ coefficients are omitted:

$$m_\nu = \begin{pmatrix} \lambda^{2b} & \lambda^b & \lambda^b \\ \lambda^b & 1 & 1 \\ \lambda^b & 1 & 1 \end{pmatrix} \frac{v_u^2}{\Lambda} \quad (\text{A, SA}) \quad , \quad (4)$$

where v_u is the VEVs of the Higgs doublet giving mass to the up quarks and all the entries are specified up to order one coefficients. If we take $v_u \sim 250$ GeV, the mass scale Λ of the heavy Majorana neutrinos turns out to be close to the unification scale, $\Lambda \sim 10^{15}$ GeV.

If b vanishes, then the light neutrino mass matrix will be structure-less and we recover the anarchical (A) picture of neutrinos¹¹⁾. In a large sample of anarchical models, generated with random coefficients, the resulting neutrino mass spectrum can exhibit either normal or inverse hierarchy. For down quarks and charged leptons we obtain a weakened hierarchy, essentially the square root than that of up quarks.

If b is positive, then the light neutrino mass matrix will be structure-less only in the (2,3) sub-sector and we get the so-called semi-anarchical (SA) models, defined by a matrix like in eq.(4) with a 23 subdeterminant generically of order 1). In this case, the neutrino mass spectrum has normal hierarchy. However, unless the (2,3) sub-determinant is accidentally suppressed, atmospheric and solar oscillation frequencies are expected to be of the same order and, in addition, the preferred solar mixing angle is small. Nevertheless, such a suppression can occur in a fraction of semi-anarchical models generated with random, order one coefficients. The real advantage over the fully anarchical scheme is represented by the suppression in U_{e3} .

Note that in all previous cases we could add a constant to $q_i^{\bar{5}}$, for example by taking $(q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) = (2 + b, 2, 2)$. This would only have the consequence to leave the top quark as the only unsuppressed mass and to decrease the resulting value of $\tan \beta = v_u/v_d$ down to $\lambda^2 m_t/m_b$. A constant shift of the charges q_i^1 might also provide a suppression of the leading ν^c mass eigenvalue, from Λ_c down to the appropriate scale Λ . One can also consider models where the 5 and $\bar{5}$ Higgs charges are different, as in the “realistic” SU(5) model of ref.¹²⁾. Also in these models the top mass could be the only one to be non-vanishing in the symmetric limit and the value of $\tan \beta$ can be adjusted.

2.1.2. F(fermions) and F(flavons) of both signs

Models with naturally large 23 splittings are obtained if we allow negative charges and, at the same time, either introduce flavons of opposite charges or stipulate that matrix elements with overall negative charge are put to zero. For example, we can assign to the fermion fields the set of F charges given by:

$$\begin{aligned} (q_1^{10}, q_2^{10}, q_3^{10}) &= (3, 2, 0) \\ (q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) &= (b, 0, 0) \quad b \geq 2a > 0 \\ (q_1^1, q_2^1, q_3^1) &= (a, -a, 0) \quad . \end{aligned} \tag{5}$$

We consider the Yukawa coupling allowed by $U(1)_F$ -neutral Higgs multiplets in the 5 and $\bar{5}$ SU(5) representations and by a pair θ and $\bar{\theta}$ of SU(5) singlets with $F = 1$ and $F = -1$, respectively. If $b = 2$ or 3, the up, down and charged lepton sectors are not

essentially different than in the SA case. Also in this case the $O(1)$ off-diagonal entry of m_l , typical of lopsided models, gives rise to a large LH mixing in the 23 block which corresponds to a large RH mixing in the d mass matrix. In the neutrino sector, after diagonalization of the charged lepton sector and after integrating out the heavy RH neutrinos we obtain the following neutrino mass matrix in the low-energy effective theory:

$$m_\nu = \begin{pmatrix} \lambda^{2b} & \lambda^b & \lambda^b \\ \lambda^b & 1 + \lambda^a \lambda'^a & 1 + \lambda^a \lambda'^a \\ \lambda^b & 1 + \lambda^a \lambda'^a & 1 + \lambda^a \lambda'^a \end{pmatrix} \frac{v_u^2}{\Lambda} \quad (\text{H}), \quad (6)$$

where λ' is given by $\langle \theta \rangle / \Lambda_c$ and Λ as before denotes the large mass scale associated to the RH neutrinos: $\Lambda \gg v_{u,d}$. The $O(1)$ elements in the 23 block are produced by combining the large LH mixing induced by the charged lepton sector and the large LH mixing in m_D . A crucial property of m_ν is that, as a result of the see-saw mechanism and of the specific $U(1)_F$ charge assignment, the determinant of the 23 block is automatically of $O(\lambda^a \lambda'^a)$ (for this the presence of negative charge values, leading to the presence of both λ and λ' is essential^{6,7}). If we take $\lambda \approx \lambda'$, it is easy to verify that the eigenvalues of m_ν satisfy the relations:

$$m_1 : m_2 : m_3 = \lambda^{2(b-a)} : \lambda^{2a} : 1 \quad . \quad (7)$$

The atmospheric neutrino oscillations require $m_3^2 \sim 10^{-3} \text{ eV}^2$. The squared mass difference between the lightest states is of $O(\lambda^{4a}) m_3^2$, not far from the LA solution to the solar neutrino problem if we choose $a = 1$. In general U_{e3} is non-vanishing, of $O(\lambda^b)$. Finally, beyond the large mixing in the 23 sector, m_ν provides a mixing angle $\theta_{12} \sim \lambda^{b-2a}$ in the 12 sector. When $b = 2a$, as for instance in the case $b = 2$ and $a = 1$, the LA solution can be reproduced and the resulting neutrino spectrum is hierarchical (H).

Alternatively, an inversely hierarchical (IH) spectrum can be obtained by choosing:

$$\begin{aligned} (q_1^{10}, q_2^{10}, q_3^{10}) &= (3, 2, 0) \\ (q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) &= (1, -1, -1) \\ (q_1^1, q_2^1, q_3^1) &= (-1, 1, 0) \\ (q_{H_u}, q_{H_d}) &= (0, 1) \quad . \end{aligned} \quad (8)$$

Due to the non-vanishing charge of the H_d Higgs doublet, in the charged lepton sector we recover the same pattern previously discussed. The light neutrino mass matrix is given by:

$$m_\nu = \begin{pmatrix} \lambda^2 & 1 & 1 \\ 1 & \lambda'^2 & \lambda'^2 \\ 1 & \lambda'^2 & \lambda'^2 \end{pmatrix} \quad (\text{IH}) \quad . \quad (9)$$

The ratio between the solar and atmospheric oscillation frequencies is not directly related to the sub-determinant of the block 23, in this case.

Table 1: Models and their flavour charges.

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

A representative set of models is listed in table 2. Note that in some cases the charges for Ψ_{10} have been changed from (3, 2, 0) (our reference values in eqs. (2), (5), and (8)) to (6, 4, 0) or (5, 3, 0). These values are a posteriori better suited to reproduce the moderate level of hierarchy implied by the present neutrino oscillation data. Since the neutrino mixing parameters are completely independent on the 10 charges, this change is only important for a better fit to quark and charged lepton masses and mixings once a rather large value of λ is derived from the neutrino data. The hierarchical and the inversely hierarchical models may come into several varieties depending on the number and the charge of the flavour symmetry breaking (FSB) parameters. Above we have considered the case of two (II) oppositely charged flavons with symmetry breaking parameters λ and λ' . It may be noticed that the presence of two multiplets θ and $\bar{\theta}$ with opposite F charges could hardly be reconciled, without adding extra structure to the model, with a large common VEV for these fields, due to possible analytic terms of the kind $(\theta\bar{\theta})^n$ in the superpotential. Therefore it is instructive to explore the consequences of allowing only the negatively charged $\bar{\theta}$ field in the theory, case I. In case I, it is impossible to compensate negative F charges in the Yukawa couplings and the corresponding entries in the neutrino mass matrices

vanish. Eventually these zeroes are filled by small contributions, arising, for instance, from the diagonalization of the charged lepton sector or from the transformations needed to make the kinetic terms canonical.

Another important ingredient is represented by the see-saw mechanism ³⁾. Hierarchical models and semi-anarchical models have similar charges in the $(10, \bar{5})$ sectors and, in the absence of the see-saw mechanism, they would give rise to similar results. Even when the results are expected to be independent from the charges of the RH neutrinos, as it is the case for the anarchical and semi-anarchical models, the see-saw mechanism can induce some sizeable effect in a statistical analysis. For this reason, for each type of model, but the normal-hierarchical ones (the mechanism for the 23 sub-determinant suppression is in fact based on the see-saw mechanism), it is interesting to study the case where RH neutrinos are present and the see-saw contribution is the dominant one (SS) and the case where they are absent and the mass matrix is saturated by the non-renormalizable contribution (NOSS).

With this classification in mind, we can distinguish the following type of models, all supported by specific choices of U(1) charges: A_{SS} , A_{NOSS} , SA_{SS} , SA_{NOSS} , $H_{(SS,I)}$, $H_{(SS,II)}$, $IH_{(SS,I)}$, $IH_{(SS,II)}$, $IH_{(NOSS,I)}$ and $IH_{(NOSS,II)}$.

It is interesting to quantify the ability of each model in reproducing the observed oscillation parameters. For anarchy, it has been observed that random generated, order-one entries of the neutrino mass matrices (in appropriate units), correctly fit the experimental data with a success rate of few percent. It is natural to extend this analysis to include also the other models based on $SU(5) \times U(1)$ ¹⁾, which have mass matrix elements defined up to order-one dimensionless coefficients y_{ij} (see eq. 1). For each model, successful points in parameter space are selected by asking that the four observable quantities $O_1 = r \equiv \Delta m_{12}^2 / |\Delta m_{23}^2|$, $O_2 = \tan^2 \theta_{12}$, $O_3 = |U_{e3}| \equiv |\sin \theta_{13}|$ and $O_4 = \tan^2 \theta_{23}$ fall in the approximately 3σ allowed ranges ¹³⁾:

$$\begin{aligned} 0.018 < r < 0.053 \\ |U_{e3}| < 0.23 \\ 0.30 < \tan^2 \theta_{12} < 0.64 \\ 0.45 < \tan^2 \theta_{23} < 2.57 \end{aligned} \tag{10}$$

The coefficients y_{ij} of the neutrino sector are random complex numbers with absolute values and phases uniformly distributed in intervals $\mathcal{I} = [0.5, 2]$ and $[0, 2\pi]$ respectively. The dependence of the results on these choices can be estimated by varying \mathcal{I} . For each model an optimization procedure selects the value of the flavour symmetry breaking parameter $\lambda = \lambda'$ that maximizes the success rate. The success rates are displayed in figs. 1 and 2, separately for the SS and NOSS cases. The two sets of models have been individually normalized to give a total rate 100. From the histograms in figs. 1 and 2 we see that normal hierarchy models (with two oppositely charged flavons H_{II}) are neatly preferred over anarchy and inverse hierarchy in the context of these $SU(5) \times U(1)$ models. In particular, in the SS case, the H_{II} models with normal

hierarchy and suppressed 23 sub-determinant are clearly preferred. Models of the type H_I are disfavoured because they tend to give $\tan\theta_{12}^2 > 1$. We recall that for the chosen charge values the H_{II} model is of the lopsided type. In the NOSS case the see-saw suppression of the 23 determinant is clearly not operative and all normal hierarchy models coincide with SA.

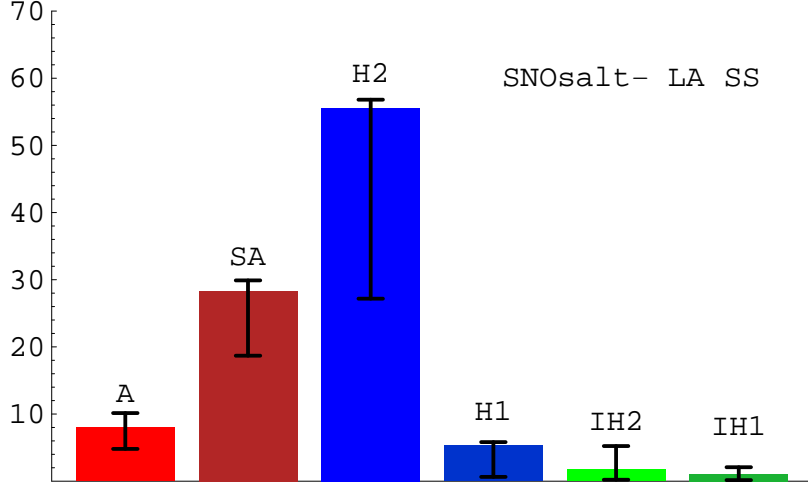


Figure 1: Relative success rates for the LA solution, with see-saw. The sum of the rates has been normalized to 100. The results correspond to the default choice $\mathcal{I} = [0.5, 2]$, and to the following values of $\lambda = \lambda'$: 0.2, 0.25, 0.35, 0.45, 0.45, 0.25 for the models A_{SS} , SA_{SS} , $H_{(SS,II)}$, $H_{(SS,I)}$, $IH_{(SS,II)}$ and $IH_{(SS,I)}$, respectively. The error bars represent the linear sum of the systematic error due to the choice of \mathcal{I} and the statistical error (see text).

An interesting question is whether the disfavouring of IH models that we find in our $SU(5) \times U(1)$ framework can be extended to a more general context. In the limit of vanishing λ and λ' the IH texture (see eq. (9)) becomes close to that of bimaximal mixing and $\theta_{13} = 0$ (actually with $r = 0$). In our $U(1)$ models $r \approx |U_{e3}| \approx |\tan^2 \theta_{12} - 1| \approx O(\lambda^2)$ (for $\lambda = \lambda'$). In particular the charged lepton mixings cannot displace too much θ_{12} from its maximal value because the small value of the electron mass forces a sufficiently large value of the relevant charges, which in turn implies that the charged lepton mixing correction to θ_{12} is small. We have already mentioned that corrections from the charged lepton sector can in principle bring the predictions of a neutrino matrix of the bimaxing type in agreement with the data and that the smallness of s_{13} induces strong constraints. In the particular setup of $U(1)_F$ models we have seen that charged lepton corrections are too small to make the solar angle sufficiently different from maximal.

In conclusion, models based on $SU(5) \times U(1)_F$ are clearly toy models that can only aim at a semiquantitative description of fermion masses. In fact only the order of magnitude of each matrix entry can be specified. However it is rather impressive that a reasonable description of fermion masses, now also including neutrino masses

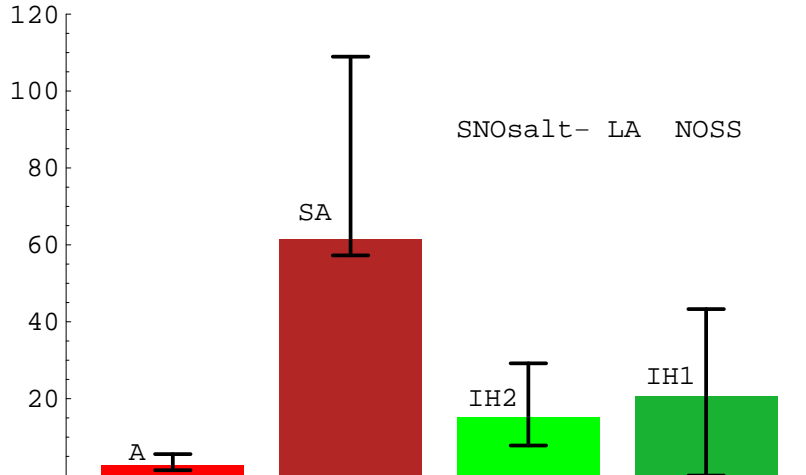


Figure 2: Relative success rates for the LA solution, without see-saw. The sum of the rates has been normalized to 100. The results correspond to the default choice $\mathcal{I} = [0.5, 2]$, and to the following values of $\lambda = \lambda'$: 0.2, 0.2, 0.25, 0.25 for the models A_{NOSS} , SA_{NOSS} , $IH_{(\text{NOSS}, \text{II})}$, and $IH_{(\text{NOSS}, \text{I})}$, respectively (in our notation there are no $H_{(\text{NOSS}, \text{I})}$, $H_{(\text{NOSS}, \text{II})}$ models). The error bars represent the linear sum of the systematic error due to the choice of \mathcal{I} and the statistical error (see text).

and mixings, can be obtained in this simple context, which is suggestive of a deeper relation between gauge and flavour quantum numbers. Moreover, all possible type of mass hierarchies can be reproduced within this framework. In a statistically based comparison, the range of r and the small upper limit on U_{e3} are sufficiently constraining to make anarchy neatly disfavoured with respect to models with built-in hierarchy. If only neutrinos are considered, one might counterargue that hierarchical models have at least one more parameter than anarchy, in our case the parameter λ . However, if one looks at quarks and leptons together, as in the GUT models that we consider, then the same parameter that plays the role of an order parameter for the CKM matrix, for example, the Cabibbo angle, can be successfully used to reproduce also the hierarchy implied by the present neutrino data. Actually it is interesting that the data now favour a moderate hierarchy, well described in terms of the moderately small Cabibbo angle.

3. Neutrino Mixings from the Charged Lepton Sector

The observed neutrino mixing matrix $U = U_e^\dagger U_\nu$, in the limit of vanishing $\sin \theta_{13} = s_{13}$, has the approximate form:

$$U = \begin{bmatrix} c & s & 0 \\ s/\sqrt{2} & -c/\sqrt{2} & 1/\sqrt{2} \\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}, \quad (11)$$

where s and c stand for $\sin \theta_{12}$ and $\cos \theta_{12}$ respectively and we took the atmospheric angle θ_{23} as exactly maximal. The effective mass matrix of light neutrinos is in general given by:

$$m_\nu = U^* m_\nu^{diag} U^\dagger . \quad (12)$$

Starting from the lagrangian basis, where all symmetries of the theory are specified, we want to investigate whether it is possible to obtain the observed mixings in a natural way from the diagonalization of the charged lepton mass matrix by U_e while U_ν is nearly diagonal. The possible deviations from maximal θ_{23} and from $s_{13} = 0$ can be omitted in eq. (11) and attributed to small effects from U_ν that will be in general not exactly zero. One might think that given the rather symmetric role of U_e and U_ν in the formula $U = U_e^\dagger U_\nu$ one way or the other should be equivalent. But we will show that this is not so. Actually now that we know that also the solar angle θ_{12} is large, this tends to clash with a small θ_{13} , in the case of mixings dominated by U_e .

In terms of U_e the charged lepton mass matrix m_e (defined as $\bar{R}m_e L$ from right-handed (R) and left-handed (L) charged lepton fields in the lagrangian basis) can be written as:

$$m_e = V_e m_e^{diag} U_e^\dagger . \quad (13)$$

Indeed $L_{diag} = U_e L$ and $R_{diag} = V_e R$ are the transformations between the lagrangian and the mass basis for the R and L fields. Assuming that $U \sim U_e^\dagger$, given that $m_e^{diag} = \text{Diag}[m_e, m_\mu, m_\tau]$ we can write:

$$m_e = V_e m_e^{diag} U = V_e \begin{bmatrix} cm_e & sm_e & 0 \\ s/\sqrt{2}m_\mu & -c/\sqrt{2}m_\mu & m_\mu/\sqrt{2} \\ -s/\sqrt{2}m_\tau & c/\sqrt{2}m_\tau & m_\tau/\sqrt{2} \end{bmatrix} . \quad (14)$$

We will come back later on the matrix V_e that determines the right-handed mixings of charged leptons. For the time being it is already interesting to consider the matrix $m_e^\dagger m_e$ which is completely fixed by U_e :

$$m_e^\dagger m_e = U_e (m_e^{diag})^2 U_e^\dagger . \quad (15)$$

Neglecting for simplicity the electron mass, we find, for $U_e^\dagger = U$:

$$m_e^\dagger m_e = U^\dagger (m_e^{diag})^2 U = \frac{1}{2}(m_\tau^2 + m_\mu^2) \begin{bmatrix} s^2 & -cs & -s(1 - 2\lambda^4) \\ -cs & c^2 & c(1 - 2\lambda^4) \\ -s(1 - 2\lambda^4) & c(1 - 2\lambda^4) & 1 \end{bmatrix} , \quad (16)$$

where we defined

$$\frac{m_\tau^2 - m_\mu^2}{m_\tau^2 + m_\mu^2} = 1 - 2\lambda^4 \quad (17)$$

so that approximately $\lambda^4 \sim m_\mu^2/m_\tau^2$. The problem with this expression for $m_e^\dagger m_e$ is that all matrix elements are of the same order and the vanishing of s_{13} as well as the

hierarchy of the eigenvalues arise from precise relations among the different matrix elements. For example, the result $s_{13} = 0$ is obtained because the eigenvector with zero eigenvalue is of the form $e_1 = (c, s, 0)^T$ and the crucial zero is present because the first two columns are proportional in eq. (16). These features are more difficult to implement in a natural way than matrices with texture zeros or with a hierarchy of matrix elements. Only if the solar angle θ_{12} is small, that is s is small, then the first row and column are nearly vanishing and s_{13} is automatically small.

Consider, for comparison, the case where we do not make the hypothesis that all the mixings are generated by the charged leptons, but rather that $U_e \sim 1$. To make the comparison more direct, let us assume that the neutrino mass spectrum is of the normal hierarchy type with dominance of m_3 : $m_\nu^{diag} \sim m_3 \text{Diag}[0, \xi^2, 1]$, where $\xi^2 = m_2/m_3$ is small and m_1 is neglected. In this case, the effective light neutrino mass matrix is given by (note the crucial transposition of U , which in eq. (11) is real, with respect to eq. (16)):

$$m_\nu = U^* m_\nu^{diag} U^\dagger \sim \frac{m_3}{2} \begin{bmatrix} s^2 \xi^2 & -cs\xi^2/\sqrt{2} & cs\xi^2/\sqrt{2} \\ -cs\xi^2/\sqrt{2} & (1 + c^2\xi^2)/2 & (1 - c^2\xi^2)/2 \\ cs\xi^2/\sqrt{2} & (1 - c^2\xi^2)/2 & (1 + c^2\xi^2)/2 \end{bmatrix}. \quad (18)$$

In this case, no matter what the value of s is, the first row and column are of order ξ^2 . By replacing terms of order ξ^2 by generic small terms of the same order, s_{13} remains of order ξ^2 . We can also replace the terms of order 1 in the 23 sector by generic order 1 quantities provided that we have a natural way of guaranteeing that the subdeterminant 23 is suppressed and remains of order ξ^2 . As well known this suppression can be naturally induced through the see-saw mechanism either by dominance of a single right-handed Majorana neutrino ¹⁴⁾ or by a lopsided ^{?)}, ¹⁵⁾ neutrino Dirac matrix. Natural realizations of this strategy have been constructed, for example, in the context of $U(1)_F$ flavour models ⁷⁾, ¹²⁾, ¹⁶⁾.

We now come back to the expression for the charged lepton mass matrix m_e in eq. (14) where the matrix V_e appears. This matrix describing the right-handed mixings of charged leptons is not related to neutrino mixings. In minimal $SU(5)$ the relation $m_e = m_d^T$ holds between the charged lepton and the down quark mass matrices. In this case V_e describes the left-handed down quark mixings: $V_e = U_d$. The CKM matrix, as well known, is given by $V_{CKM} = U_u^\dagger U_d$. Given that the quark mixing angles are small, either both U_u and U_d are nearly diagonal or they are nearly equal. Thus one possibility is that U_d is nearly diagonal. In this case, for $V_e = U_d$, m_e is approximately given by eq. (14) with $V_e \sim 1$. Neglecting the electron mass and setting $\lambda^2 = m_\mu/m_\tau$ we obtain:

$$m_e \approx m_e^{diag} U = m_\tau \begin{bmatrix} 0 & 0 & 0 \\ s/\sqrt{2}\lambda^2 & -c/\sqrt{2}\lambda^2 & \lambda^2/\sqrt{2} \\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}. \quad (19)$$

This matrix is a generalization of lopsided models with all three matrix elements in the third row of order 1 (unless s is small: for small solar angle we go back to the situation of normal lopsided models). We recall that lopsided models with the 23 and 33 matrix elements of order 1 provide a natural way to understand a large 23 mixing angle. In fact from the matrix relation

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad (20)$$

we see that in lopsided models one automatically gets a large 23 mixing from U_e . In the generalized case of eq. (19), while the natural prediction of a large 23 mixings remains, the relation $s_{13} = 0$ does not arise automatically if the entries of the matrix are replaced by generic order 1 terms in the third row and of order λ^2 in the second row. If we call v_3 the 3-vector with components of order 1 in the third row and $\lambda^2 v_\lambda$ the vector of the second row, we can easily check that to obtain $s_{13} = 0$ it is needed that both v_λ and v_3 are orthogonal to a vector of the form $(c, s, 0)$.

In democratic models all matrices U_u, U_d, U_e are nearly equal and the smallness of quark mixings arises from a compensation between U_u^\dagger and U_d . This sort of models correspond, for $V_e = U_d$, to $V_e = U_e = U^\dagger$ and a symmetric matrix m_e : $m_e = U^\dagger m_e^{diag} U$. In this case we obtain a matrix exactly equal to that in eq. (16) for $m_e^\dagger m_e$ except that squared masses are replaced by masses. As discussed in the case of eq. (16), we need fine-tuning in order to reproduce the observed hierarchy of mass and to obtain $s_{13} = 0$ unless the solar angle s is small. Note in fact, that in the democratic model of ¹⁷⁾, the vanishing of s_{13} is only accommodated but not predicted.

3.1. A Natural Class of Models

We now attempt to identify a set of conditions that make possible the construction of an explicit model where the mixing in the lepton sector is dominated by the charged lepton contribution. One obvious condition is a dynamical or a symmetry principle that forces the light neutrino mass matrix to be diagonal in the lagrangian basis. The simplest flavour symmetries cannot fulfill this requirement in a simple way. For instance, a $U(1)$ symmetry can lead to a nearly diagonal neutrino mass matrix, of the form:

$$m_\nu = \begin{bmatrix} \xi^{2p} & \xi^{p+1} & \xi^p \\ \xi^{p+1} & \xi^2 & \xi \\ \xi^p & \xi & 1 \end{bmatrix} m \quad , \quad (21)$$

where $\xi < 1$ is a $U(1)$ breaking parameter, $p \geq 1$ and all matrix elements are defined up to unknown order one coefficients. The problem with this matrix is that the ratio between the solar and the atmospheric squared mass differences, close to $1/40$, is approximately given by ξ^4 and, consequently, a large atmospheric mixing angle is

already induced by m_ν itself. If we consider a discrete symmetry like S_3 , m_ν can be of the general form:

$$m_\nu = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} m + \alpha \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} m \quad , \quad (22)$$

where α is an arbitrary parameter. In this case we need both the extra assumption $|\alpha| \ll 1$ and a specific symmetry breaking sector to lift the mass degeneracy¹⁷⁾. A stronger symmetry like $O(3)$ removes from eq. (22) the non-diagonal invariant, but requires a non-trivial symmetry breaking sector with a vacuum alignment problem in order to keep the neutrino sector diagonal while allowing large off-diagonal terms for charged leptons¹⁸⁾.

A simple, though not economical, possibility to achieve a diagonal neutrino mass matrix, is to introduce three independent $U(1)$ symmetries, one for each flavour²⁾: $F = U(1)_{F_1} \times U(1)_{F_2} \times U(1)_{F_3} \times \dots$, where F denotes the flavour symmetry group. The Higgs doublet giving mass to the up-type quarks is neutral under F . Each lepton doublet is charged under a different $U(1)$ factor, with the same charge $+1$. In the symmetric phase all neutrinos are exactly massless. Flavour symmetry breaking is obtained by non-vanishing vacuum expectation values (VEVs) of three flavon fields, also charged under a separate $U(1)$ factor, with charge -2 . In this way only diagonal neutrino mass terms are induced. If the VEVs in the flavon sector are similar, we expect neutrino masses of the same order, and the observed hierarchy between the solar and atmospheric squared mass differences requires a modest adjustment of the flavon vev's and/or of the coefficients of the lepton violating operators.

A second condition can be identified by considering a mass matrix for the charged leptons which is very close, but slightly more general than the one of eq. (19):

$$m_e = \begin{bmatrix} O(\lambda^4) & O(\lambda^4) & O(\lambda^4) \\ x_{21}\lambda^2 & x_{22}\lambda^2 & O(\lambda^2) \\ x_{31} & x_{32} & O(1) \end{bmatrix} m \quad , \quad (23)$$

where x_{ij} ($i = 2, 3$) ($j = 1, 2$) is a matrix of order one coefficients with vanishing determinant:

$$x_{21}x_{32} - x_{22}x_{31} = 0 \quad . \quad (24)$$

The eigenvalues of m_e in units of m are of order 1, λ^2 and λ^4 , as required by the charged lepton masses. Moreover, the eigenvalue of order λ^8 of $m_e^\dagger m_e$ has an eigenvector:

$$(c, s, O(\lambda^4)) \quad \frac{s}{c} = -\frac{x_{31}}{x_{32}} + O(\lambda^4) \quad . \quad (25)$$

In terms of the lepton mixing matrix $U = U_e^\dagger$, this means $\theta_{13} = O(\lambda^4)$ and θ_{12} large, if $x_{31} \approx x_{32}$. When the remaining, unspecified parameters in m_e are all of order

one, also the atmospheric mixing angle θ_{23} is large. Notice that, by neglecting $O(\lambda^4)$ terms, the following relation holds for the mass matrix m_e/m (cfr. eq. (20)):

$$\begin{bmatrix} 0 & 0 & 0 \\ x_{21}\lambda^2 & x_{22}\lambda^2 & O(\lambda^2) \\ x_{31} & x_{32} & O(1) \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sqrt{x_{21}^2 + x_{22}^2}\lambda^2 & O(\lambda^2) \\ 0 & \sqrt{x_{31}^2 + x_{32}^2} & O(1) \end{bmatrix}, \quad (26)$$

where

$$\frac{s}{c} = -\frac{x_{31}}{x_{32}}. \quad (27)$$

Therefore the natural parametrization of the unitary matrix U_e that diagonalizes $m_e^\dagger m_e$ in this approximation is:

$$U_e = U_{12}^e U_{23}^e, \quad (28)$$

where U_{ij}^e refers to unitary transformation in the ij plane. Using $U = U_e^\dagger$, we automatically find the leptonic mixing matrix in the standard parametrization $U = U_{23}U_{13}U_{12}$ (neglecting phases), with $U_{23} = U_{23}^{\dagger e}$, $U_{13} = 1$ and $U_{12} = U_{12}^{\dagger e}$. Had we used the standard parametrization also for U_e , we would have found three non-vanishing rotation angles θ_{ij}^e with non-trivial relations in order to reproduce $\theta_{13} = 0$.

This successful pattern of m_e , eq. (23), has two features. The first one is the hierarchy between the rows. It is not difficult to obtain this in a natural way. For instance, we can require a $U(1)$ flavour symmetry acting non-trivially only on the right-handed charged leptons, thus producing the required suppressions of the first and second rows. The second one is the vanishing determinant condition of eq. (24). We can easily reproduce this condition by exploiting a see-saw mechanism operating in the charged lepton sector.

To show this we add to the field content of the standard model additional vector-like fermion pairs (L_a, L_a^c) ($a = 1, \dots, n$) of $SU(2)$ doublets, with hypercharges $Y = (-1/2, +1/2)$. The Lagrangian in the charged lepton sector reads:

$$\mathcal{L} = \text{kinetic terms} + \eta_{ij} e_i^c l_j h_d + \lambda_{ia} e_i^c L_a h_d + \mu_{aj} L_a^c l_j + M_a L_a^c L_a + h.c. \quad (29)$$

where l_i and e_i^c ($i = 1, 2, 3$) are the standard model leptons, doublet and singlet under $SU(2)$, respectively, and h_d denotes the Higgs doublet. We assume a diagonal mass matrix for the extra fields. We expect $M_a, \mu_{aj} \gg \langle h_d \rangle$ and in this regime there are heavy fermions that can be integrated out to produce a low-energy effective Lagrangian. The heavy combinations are L_a^c and

$$L_a + \frac{\mu_{aj}}{M_a} l_j \quad (a = 1, \dots, n) \quad (30)$$

These fields are set to zero by the equations of motion in the static limit and we should express all remaining fermions in term of the three combinations that are orthogonal

to those in eq. (30), and which we identify with the light degrees of freedom. To illustrate our point it is sufficient to work in the regime $|\mu_{aj}| < |M_a|$ and expand the relevant quantities at first order in $|\mu_{aj}/M_a|$. To this approximation the light lepton doublets l'_i are:

$$l'_i = l_i - \frac{\mu_{ai}}{M_a} L_a \quad . \quad (31)$$

The effective lagrangian reads:

$$\mathcal{L} = \text{kinetic terms} + (\eta_{ij} - \frac{\lambda_{ia}\mu_{aj}}{M_a}) e_i^c l'_j h_d + h.c. \quad , \quad (32)$$

and the mass matrix for the charged leptons is:

$$m_e = \langle h_d \rangle (\eta_{ij} - \frac{\lambda_{ia}\mu_{aj}}{M_a}) \quad . \quad (33)$$

This result is analogous to what obtained in the neutrino sector from the see-saw mechanism. There is a term in m_e coming from the exchange of the heavy fields (L_a, L_a^c) , which play the role of the right-handed neutrinos, and there is another term that comes from a single operator and that cannot be interpreted as due to the exchange of heavy modes. In the regime $1 > |\mu/M| > |\eta/\lambda|$ the “see-saw” contribution dominates. Moreover, if the lower left block in m_e is dominated by a single exchange, for instance by (L_1, L_1^c) , then

$$\begin{bmatrix} m_{e21} & m_{e22} \\ m_{e31} & m_{e32} \end{bmatrix} = \frac{\langle h_d \rangle}{M_1} \begin{bmatrix} \lambda_{21}\mu_{11} & \lambda_{21}\mu_{12} \\ \lambda_{31}\mu_{11} & \lambda_{31}\mu_{12} \end{bmatrix} \quad , \quad (34)$$

and the condition of vanishing determinant in eq. (24) is automatically satisfied.

Additional vector-like leptons are required by several extensions of the standard model. For instance, a grand unified theory based on the E_6 gauge symmetry group with three generations of matter fields described by three 27 representations of E_6 , includes, beyond the standard model fermions, two $SU(5)$ singlets and an $SU(5)$ vector-like $(5, \bar{5})$ pair per each generation. In such a model a “see-saw” mechanism induced by the exchange of heavy $(5, \bar{5})$ fields is not an option, but a necessary ingredient to recover the correct number of light degrees of freedom. We should still show that it is possible to combine the above conditions in a natural and consistent framework. In ref.^{?)} we have presented, as an existence proof, a supersymmetric $SU(5)$ grand unified model possessing a flavour symmetry $F=U(1)_{F_0} \times U(1)_{F_1} \times U(1)_{F_2} \times U(1)_{F_3}$. The first $U(1)_{F_0}$ factor is responsible for the hierarchy of masses and mixing angles in the up-type quark sector as well as for the hierarchy between the rows in the charged lepton mass matrix. The remaining part of F guarantees a diagonal neutrino mass matrix and, at the same time, dominance of a single heavy $(5, \bar{5})$ pair in the lower left block of m_e . Notice that, at variance with most of the other existing models¹⁹⁾,

this framework predicts a small value for θ_{13} , of order λ^4 which is at the border of sensitivity of future neutrino factories.

3.2. Corrections to Bimixing from U_e

Even when the neutrino mass matrix U_ν is not diagonal in the lagrangian basis, the contribution from the charged lepton sector can be relevant or even crucial to reproduce the observed mixing pattern. An important example arises if the neutrino matrix U_ν instead of being taken as nearly diagonal, is instead assumed of a particularly simple form, like for bimixing:

$$U_\nu = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{bmatrix} . \quad (35)$$

This configuration can be obtained, for instance, in inverse hierarchy models with a $L_e - L_\mu - L_\tau$ U(1) symmetry, which predicts maximal θ_{12}^ν , large θ_{23}^ν , vanishing θ_{13}^ν and $\Delta m_{sol}^2 = 0$. After the breaking of this symmetry, the degeneracy between the first two neutrino generations is lifted and the small observed value of Δm_{sol}^2 can be easily reproduced. Due to the small symmetry breaking parameters, the mixing angles in eq. (35) also receive corrections, whose magnitude turns out ²⁰⁾ to be controlled by $\Delta m_{sol}^2 / \Delta m_{atm}^2$: $\theta_{13}^\nu \lesssim 1 - \tan^2 \theta_{12}^\nu \sim \Delta m_{sol}^2 / (2\Delta m_{atm}^2) \sim 0.01$. These corrections are too small to account for the measured value of the solar angle. Thus, an important contribution from U_e is necessary to reconcile bimixing with observation.

In this section we will reconsider the question of whether the observed pattern can result from the corrections induced by the charged lepton sector. Though not automatic, this appears to be at present a rather natural possibility ²¹⁾ - see also the recent detailed analysis of Ref. ²²⁾. Our aim is to investigate the impact of planned experimental improvements, in particular those on $|U_{e3}|$, on bimixing models. To this purpose it is useful to adopt a convenient parametrization of mixing angles and phases. Let us define

$$\tilde{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad (36)$$

where all the mixing angles belong to the first quadrant and δ to $[0, 2\pi]$. The standard parameterization for U reads: $U = \tilde{U} \times$ a diagonal $U(3)$ matrix accounting for the two Majorana phases of neutrinos (the overall phase is not physical). Since in the following discussion we are not interested in the Majorana phases, we will focus our attention on \tilde{U} .

It would be appealing to take the parameterization (36) separately for U_e and U_ν , by writing s_{12} , s_{12}^e , s_{12}^ν etc to distinguish the mixings of the U , U_e and U_ν matrices,

respectively. However, as discussed in ref. ²⁾, even disregarding Majorana phases, U is not just determined in terms of \tilde{U}_e and \tilde{U}_ν , with the latter defined to be of the form (36). The reason is that, by means of field redefinitions U_e and U_ν can be separately but *not simultaneously* written respectively as \tilde{U}_e and $\tilde{U}_\nu \times$ a diagonal $U(3)$ matrix. Without loss of generality we can adopt the following form for U :

$$U = U_e^\dagger U_\nu = \underbrace{\tilde{U}_e^\dagger \text{diag}(-e^{-i(\alpha_1+\alpha_2)}, -e^{-i\alpha_2}, 1)}_{=\bar{U}} \tilde{U}_\nu \times \text{phases} \quad (37)$$

where $\tilde{U}_e, \tilde{U}_\nu$ have the form (36), the phases α_1, α_2 run from 0 to 2π and we have introduced two minus signs in the diagonal matrix for later convenience. This expression for \bar{U} is not due to the Majorana nature of neutrinos and a similar result would also hold for quarks.

Assume now that \tilde{U}_ν corresponds to bimaxing: $s_{13}^\nu = 0, s_{12}^\nu = c_{12}^\nu = 1/\sqrt{2}$ and $s_{23}^\nu = c_{23}^\nu = 1/\sqrt{2}$. Clearly, our discussion holds true irrespectively of the light neutrino spectrum. It is anyway instructive to explicitate the mass matrices, e.g. in the case of inverted hierarchy

$$m_\nu = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \frac{\Delta m_{atm}^2}{\sqrt{2}}, \quad m_e = V_e \begin{pmatrix} m_e e^{-i(\alpha_1+\alpha_2)} & -s_{12}^e m_e e^{-i\alpha_2} & -s_{13}^e m_e e^{i\delta_e} \\ s_{12}^e m_\mu e^{-i(\alpha_1+\alpha_2)} & m_\mu e^{-i\alpha_2} & -s_{23}^e m_\mu \\ s_{13}^e m_\tau e^{-i(\alpha_1+\alpha_2+\delta_e)} & s_{23}^e m_\tau e^{-i\alpha_2} & m_\tau \end{pmatrix} \quad (38)$$

where we have set $\Delta m_{sol}^2 = 0$ since, as already mentioned, the corrections induced by setting it to the measured value are negligible in the present discussion.

We then expand \bar{U} of eq. (37) at first order in the small mixings of $\tilde{U}_e, s_{12}^e, s_{13}^e$ and s_{23}^e *:

$$\begin{aligned} \bar{U}_{11} &= -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} - \frac{s_{12}^e e^{-i\alpha_2} + s_{13}^e e^{i\delta_e}}{2} \\ \bar{U}_{12} &= -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s_{12}^e e^{-i\alpha_2} + s_{13}^e e^{i\delta_e}}{2} \\ \bar{U}_{13} &= \frac{s_{12}^e e^{-i\alpha_2} - s_{13}^e e^{i\delta_e}}{\sqrt{2}} \\ \bar{U}_{23} &= -e^{-i\alpha_2} \frac{1 + s_{23}^e e^{i\alpha_2}}{\sqrt{2}} \\ \bar{U}_{33} &= \frac{1 - s_{23}^e e^{-i\alpha_2}}{\sqrt{2}}. \end{aligned} \quad (39)$$

The smallness of the observed s_{13} implies that both s_{12}^e and s_{13}^e must be at most of order s_{13} . As a consequence, the amount of the deviation of s_{12} from $1/\sqrt{2}$ is limited

*To this approximation any ordering of the three small rotations in U_e gives exactly the same results, and our conclusions are independent on the adopted parametrization.

from the fact that it is generically of the same order as s_{13} . Note that, instead, the deviation of the atmospheric angle s_{23} from $1/\sqrt{2}$ is of second order in s_{12}^e and s_{13}^e , so that it is natural to expect a smaller deviation as observed. From eqs. (39) we obtain the following explicit expressions for the observable quantities:

$$\tan^2 \theta_{23} = 1 + 4s_{23}^e \cos(\alpha_2) \quad (40)$$

$$\delta_{sol} \equiv 1 - \tan^2 \theta_{12} = 2\sqrt{2}(s_{12}^e \cos(\alpha_1) + s_{13}^e \cos(\delta_e + \alpha_2 + \alpha_1)) \quad (41)$$

$$|U_{e3}| = \frac{1}{\sqrt{2}}(s_{12}^e{}^2 + s_{13}^e{}^2 - 2\cos(\delta_e + \alpha_2)s^e s_{13}^e)^{1/2} \quad (42)$$

$$\tan \delta = \frac{s_{12}^e \sin(\alpha_1) - s_{13}^e \sin(\delta_e + \alpha_2 + \alpha_1)}{s_{12}^e \cos(\alpha_1) - s_{13}^e \cos(\delta_e + \alpha_2 + \alpha_1)}, \quad (43)$$

to be compared with the experimental data. According to ¹³⁾ the $3\text{-}\sigma$ windows are $|U_{e3}| \leq 0.23$ and $0.36 \leq \delta_{sol} \leq 0.70$.

Notice that the sign of δ_{sol} is not necessarily positive, so that only a part (say half) of the parameter space in principle allowed for the phases is selected. With the correction to δ_{sol} going in the good direction, one roughly expects $|U_{e3}| \sim \delta_{sol}/4 \approx 0.1 - 0.2$. Hence, at present it is not excluded that charged lepton mixing can transform a bimixing configuration into a realistic one but there are constraints and, in order to minimize the impact of those constraints, $|U_{e3}|$ must be within a factor of 2 from its present upper limit. On the other hand, an upper limit on $|U_{e3}|$ smaller than $\delta_{sol}/4$ would start requiring a fine-tuning. Indeed, in order to reduce $|U_{e3}|$ significantly below $0.1 - 0.2$ a cancellation must be at work in eq. (42), namely $\delta_e + \alpha_2$ should be close to 0 or 2π and s_{12}^e and s_{13}^e should be of comparable magnitude. In addition, to end up with the largest possible $\delta_{sol}/4$, eq. (41) would also suggest a small value for α_1 .

The above considerations can be made quantitative by showing, for different upper bounds on $|U_{e3}|$, the points of the plane $[s_{12}^e, s_{13}^e]$ which are compatible with the present 3σ window for the solar angle. This is shown in fig. 3, where the three plots correspond to different choices for α_1 . A point in the plane $[s_{12}^e, s_{13}^e]$ is excluded if there is no value of $\alpha_2 + \delta_e$ for which (42) and (41) agree with experiment. Regions in white are those excluded by the present bound on $|U_{e3}|$. With increasingly stronger bounds on $|U_{e3}|$, the allowed regions, indicated in the plots with increasingly darkness, get considerably shrunk. For $|U_{e3}| \leq 0.05$ only $|\alpha_1| < \pi/2$ is allowed. Notice also that at present the two most natural possibilities $s_{12}^e \gg s_{13}^e$ and $s_{12}^e \ll s_{13}^e$ are allowed but, with $|U_{e3}| < 0.1$, they are significantly constrained and with $|U_{e3}| \leq 0.05$ ruled out completely. Below the latter value for $|U_{e3}|$, a high level of degeneracy between s_{12}^e and s_{13}^e together with a small value for α_1 and $\delta_e + \alpha_2$ are required.

Summarising, planned improvements in the sensitivity to $|U_{e3}|$ - which could reach the 0.05 level, could have a crucial impact on bimixing models. They could either disfavour it as unnatural (in the sense that a dynamical principle or a symmetry

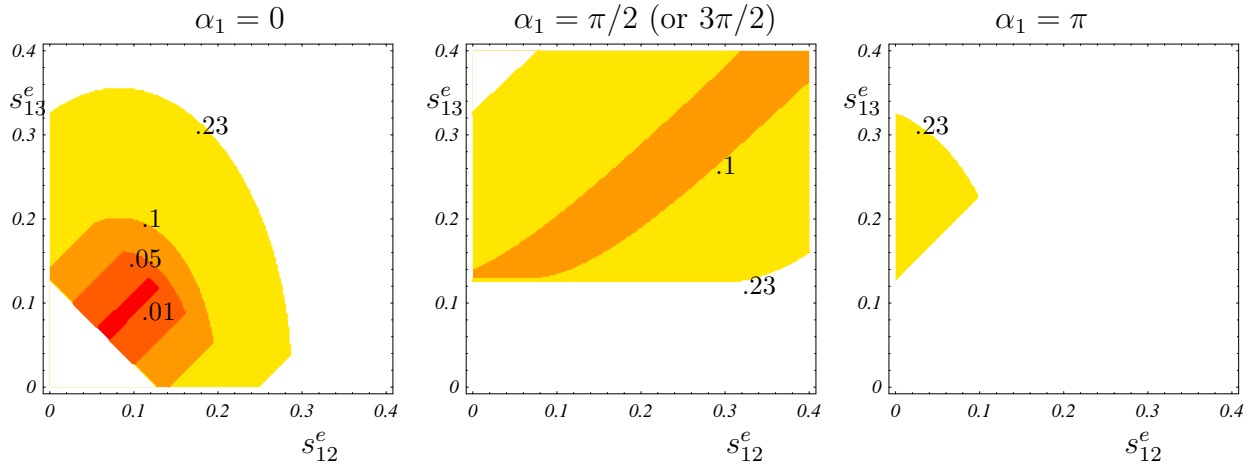


Figure 3: Taking an upper bound on $|U_{e3}|$ respectively equal to 0.23, 0.1, 0.05, 0.01, we show (from yellow to red) the allowed regions of the plane $[s_{12}^e, s_{13}^e]$. Each plot is obtained by setting α_1 to a particular value, while leaving $\alpha_2 + \delta_e$ free. We keep the present 3σ window for δ_{sol} ¹³).

acting also on the charged lepton mass matrix would have to be invoked) or, if $|U_{e3}|$ were to be found, support bimixing models.

4. References

- 1) G. Altarelli, F. Feruglio and I. Masina, JHEP **0301** (2003) 035 [arXiv:hep-ph/0210342].
- 2) G. Altarelli, F. Feruglio and I. Masina, hep-ph/0402155.
- 3) T. Yanagida, in *Proc. of the Workshop on Unified Theory and Baryon Number in the Universe*, KEK, March 1979; M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, Stony Brook, Sept 1979; S. L. Glashow, in “Quarks and Leptons”, Cargèse, ed. M. Lévy et al., Plenum, 1980 New York, p. 707; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.
- 4) C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147, 277 (1979).
- 5) Y. Kawamura, Prog. Theor. Phys. **105** (2001) 999 [arXiv:hep-ph/0012125]. G. Altarelli and F. Feruglio, Phys. Lett. B **511** (2001) 257 [arXiv:hep-ph/0102301]. L. J. Hall and Y. Nomura, Phys. Rev. D **64** (2001) 055003 [arXiv:hep-ph/0103125]. A. Hebecker and J. March-Russell, Nucl. Phys. B **613** (2001) 3 [arXiv:hep-ph/0106166]. T. Asaka, W. Buchmüller and L. Covi, Phys. Lett. B **523** (2001) 199 [arXiv:hep-ph/0108021]. L. J. Hall, Y. Nomura, T. Okui and D. R. Smith, Phys. Rev. D **65** (2002) 035008 [arXiv:hep-ph/0108071]. T. Asaka, W. Buchmüller and L. Covi, Nucl. Phys. B **648** (2003) 231 [arXiv:hep-ph/0209144]. R. Dermisek and A. Mafi, Phys. Rev. D **65** (2002)

- 055002 [arXiv:hep-ph/0108139];
H. D. Kim and S. Raby, JHEP **0301** (2003) 056 [arXiv:hep-ph/0212348].
A. Hebecker and M. Ratz, Nucl. Phys. B **670** (2003) 3
[arXiv:hep-ph/0306049].
- 6) G. Altarelli and F. Feruglio, Phys. Lett. B **439**, 112 (1998); G. Altarelli and F. Feruglio, JHEP **9811**, 021 (1998).
 - 7) G. Altarelli and F. Feruglio, Phys. Lett. B **451**, 388 (1999).
 - 8) H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
 - 9) J. Bijnens and C. Wetterich, Nucl. Phys. B **147**, 292 (1987); M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **398**, 319 (1993); Nucl. Phys. B **420**, 468 (1994); L. E. Ibanez and G. G. Ross, Phys. Lett. B **332**, 100 (1994); P. Binetruy and P. Ramond, Phys. Lett. B **350**, 49 (1995); E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. B **356**, 45 (1995); G. K. Leontaris and J. Rizos, Nucl. Phys. B **567** (2000) 32; J. M. Mira, E. Nardi and D. A. Restrepo, Phys. Rev. D **62**, 016002 (2000).
 - 10) P. Binetruy, S. Lavignac and P. Ramond, Nucl. Phys. B **477**, 353 (1996); P. Binetruy, S. Lavignac, S. T. Petcov and P. Ramond, Nucl. Phys. B **496**, 3 (1997); Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B **396**, 150 (1997); M. Jezabek and Y. Sumino, Phys. Lett. B **440**, 327 (1998); Y. Grossman, Y. Nir and Y. Shadmi, JHEP **9810**, 007 (1998); W. Buchmuller and T. Yanagida, Phys. Lett. B **445**, 399 (1999); C. D. Froggatt, M. Gibson and H. B. Nielsen, Phys. Lett. B **446**, 256 (1999); K. Choi, K. Hwang and E. J. Chun, Phys. Rev. D **60**, 031301 (1999); S. Lola and G. G. Ross, Nucl. Phys. B **553**, 81 (1999); Y. Nir and Y. Shadmi, JHEP **9905**, 023 (1999); G. Altarelli, F. Feruglio and I. Masina, Phys. Lett. B **472**, 382 (2000); B. Stech, Phys. Rev. D **62**, 093019 (2000); S. F. King and M. Oliveira, Phys. Rev. D **63**, 095004 (2001); M. Tanimoto, Phys. Lett. B **501**, 231 (2001); M. Bando and N. Maekawa, Prog. Theor. Phys. **106**, 1255 (2001). H. B. Nielsen and Y. Takanishi, Phys. Scripta **T93**, 44 (2001).
 - 11) L. J. Hall, H. Murayama and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000); N. Haba and H. Murayama, Phys. Rev. D **63**, 053010 (2001); M. S. Berger and K. Siyeon, Phys. Rev. D **63**, 057302 (2001); M. Hirsch, arXiv:hep-ph/0102102; M. Hirsch and S. F. King, Phys. Lett. B **516** (2001) 103 [arXiv:hep-ph/0102103]; F. Vissani, Phys. Lett. B **508**, 79 (2001). A. de Gouvea and H. Murayama, arXiv:hep-ph/0301050; J. R. Espinosa, arXiv:hep-ph/0306019.
 - 12) G. Altarelli, F. Feruglio and I. Masina, JHEP **0011**, 040 (2000);
 - 13) M. Maltoni, T. Schwetz, M.A. Tortola, J.W.F. Valle, Phys.Rev. D **68** (2003) 113010, hep-ph/0309130. See also: J. N. Bahcall and C. Pena-Garay, JHEP **0311** (2003) 004 [arXiv:hep-ph/0305159]; M. C. Gonzalez-Garcia and C. Pena-Garay, Phys. Rev. D **68** (2003) 093003 [arXiv:hep-ph/0306001]; P. Aliani,

- V. Antonelli, M. Picariello and E. Torrente-Lujan, arXiv:hep-ph/0309156; A. Bandyopadhyay, S. Choubey, S. Goswami, S. T. Petcov and D. P. Roy, arXiv:hep-ph/0309174; G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, eConf **C030626** (2003) THAT05 [arXiv:hep-ph/0310012].
- 14) A.Yu. Smirnov, Phys. Rev. **D48** (1993) 3264, hep-ph/9304205; S. F. King, Phys. Lett. **B439** (1998) 350, hep-ph/9806440; S. Davidson and S. F. King, Phys. Lett. **B445** (1998) 191, hep-ph/9808296; R. Barbieri, L.J. Hall, A. Strumia, Phys. Lett. **B445** (1999) 407, hep-ph/9808333; Q. Shafi and Z. Tavartkiladze, Phys. Lett. **B 451** (1999) 129, hep-ph/9901243; R. Barbieri, P. Creminelli, A. Romanino, Nucl. Phys. **B559** (1999) 17, hep-ph/9903460; S. F. King, Nucl. Phys. **B562**, (1999) 57, hep-ph/9904210; S. F. King, Nucl. Phys. **B576**, (2000) 85, hep-ph/9912492.
 - 15) C.H. Albright and S.M. Barr, Phys. Rev. **D58** (1998) 013002, hep-ph/9712488; C.H. Albright, K.S. Babu and S.M. Barr, Phys. Rev. Lett. **81** (1998) 1167, hep-ph/9802314; P.H. Frampton and A. Rasin, Phys. Lett. **B478** (2000) 424, hep-ph/9910522.
 - 16) J. Sato and T. Yanagida, Phys. Lett. **B430** (1998) 127, hep-ph/9710516; N. Irges, S. Lavignac and P. Ramond, Phys. Rev. **D58** (1998) 035003, hep-ph/9802334; J.K. Elwood, N. Irges and P. Ramond, Phys. Rev. Lett. **81** (1998) 5064, hep-ph/9807228; Z. Berezhiani and A. Rossi, JHEP **9903** (1999) 002, hep-ph/9811447; M. Bando and T. Kugo, Prog. Theor. Phys. **101** (1999) 1313, hep-ph/9902204;
 - 17) H. Fritzsch, Z.Z. Xing, Phys. Lett. **B372** (1996) 265; Phys. Lett. **B440** (1998) 313; Prog. Part. Nucl. Phys. **45** (2000) 1; M. Fukugita, M. Tanimoto, T. Yanagida, Phys. Rev. **D57** (1998) 4429; Phys. Rev. **D59** (1999) 113016, hep-ph/9809554 ; M. Tanimoto, T. Watari, T. Yanagida, Phys. Lett. **B461** (1999) 345; S.K. Kang and C.S. Kim, Phys. Rev. **D59** (1999) 091302; M. Tanimoto, Phys. Lett. **B483** (2000) 417; N. Haba, Y. Matsui, N. Okamura, T. Suzuki, Phys. Lett. **B489** (2000) 184; Y. Koide and A. Ghosal, Phys. Lett. **B488** (2000) 344; E.K. Akhmedov, G.C. Branco, F.R. Joaquim and J.I. Silva-Marcos, Phys. Lett. **B498** (2001) 237. For models based on discrete symmetries see also: P.F. Harrison, D.H. Perkins and W.G. Scott, Phys. Lett. **B530** (2002) 167; P.F. Harrison and W.G. Scott, hep-ph/0203209; R. Dermisek, hep-ph/0312206.
 - 18) R. Barbieri, L.J. Hall, G.L. Kane and G.G. Ross, hep-ph/9901228.
 - 19) See for instance: G. Altarelli and F. Feruglio, arXiv:hep-ph/0306265, in the proceedings of 10th International Workshop on Neutrino Telescopes, Venice, Italy, 11-14 Mar 2003, vol. 1, pp. 237-277.
 - 20) F. Feruglio, A. Strumia and F. Vissani, Nucl. Phys. B **637** (2002) 345 [Addendum-ibid. B **659** (2003) 359] hep-ph/0201291; S. Lavignac, I. Masina,

- C.A. Savoy, Nucl. Phys. **B633** (2002) 139, hep-ph/0202086.
- 21) S.F. King, N. Nimai Singh, Nucl. Phys. **B596** (2001) 81, hep-ph/0007243;
S.F. King, JHEP **0209** (2002) 011, hep-ph/0204360; R. Barbieri, T. Hambye,
A. Romanino, JHEP **0303** (2003) 017, hep-ph/0302118.
- 22) P.H. Frampton, S.T. Petcov, W. Rodejohann, hep-ph/0401206.